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MARINE GEODETIC CONTROL FOR GEOIDAL PROFILE MAPPING ACROSS THE PUERTO RICAN TRENCH

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16. Abstract

The objective of this investigation was to establish a marine geodetic control for the northern end of the geoidal profile mapping experiment across the Puerto Rican Trench. The marine geodetic control was established by determining the three-dimensional geodetic coordinates of the four ocean-bottom mounted acoustic transponders. The data reduction techniques employed and two new analytical processes involved are described. Before applying these new analytical techniques to the field data, they were tested with simulated data and proven to be effective in theory as well as in practice.

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ABSTRACT

The objective of this investigation was to establish a marine geodetic control for the northern end of the geoidal profile mapping experiment across the Puerto Rican Trench. The experiment was conducted under the sponsorship of NASA/Wallops Station, using the Apollo ship VANGUARD. This investigation is a sequal to a July, 1971 report - Planning and Conducting an Ocean Surface Mapping Experiment Using Apollo Ship Instrumentation.

The marine geodetic control was successfully established by determining the three-dimensional geodetic coordinates of the four ocean-bottom mounted acoustic transponders. The data reduction techniques employed and two new analytical processes involved are described. Before applying these new analytical techniques to the field data, they were tested with simulated data and proven to be effective in theory as well as in practice. The simulation study also provided invaluable guidelines into the selection of the most desirable sets of data from the experiment. The computed geographic location of the control point, as reported, is dependent on the Ship's Inertial Navigation System (SINS). Because SINS coordinates are not truly geodetic, determination of the absolute position (i.e., relative to earth's center of mass) of the marine geodetic control will have to await the incorporation of the results from satellite observations conducted during the experiment.

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MARINE GEODETIC CONTROL FOR GEOIDAL PROFILE MAPPING ACROSS THE PUERTO RICAN TRENCH

by

D. M. Fubara and A. G. Mourad

1.0 INTRODUCTION

This report on a marine geodetic experiment conducted in collaboration with NASA personnel and others describes the data-reduction techniques employed and the analytical considerations involved in the determination of three-dimensional coordinates to establish a geodetic control station in the Puerto Rican Trench Region. This report is a sequal to a prior Battelle report(5) - Planning and Conducting an Ocean Surface Mapping Experiment Using Apollo Ship Instrumentation.

Two techniques developed at Battelle-Columbus for geodetic location of ocean-bottom-mounted acoustic transponders were used during this investigation: LEast Squares Sequential Adjustment (LESSA) and FIxation from COplanar Ranges (FIXCOR). The simulated data investigation conducted has conclusively proved that LESSA and FIXCOR are theoretically and practically workable, and effective in meeting the objectives for which they were developed.

LESSA and FIXCOR were applied to acoustic and navigational data acquired in the Puerto Rican Trench Geodetic Experiment conducted under NASA/Wallops Station sponsorship, between June 25 and July 5, 1970, using the Apollo tracking ship USNS VANGUARD. The overall objective of the experiment was to determine the geoidal profile across the Puerto Rican Trench (for details see Reference 5).

In brief review, the experiment was conducted along the track depicted in Figure 1. During the experiment, SRN-9 Doppler satellite observations were made as often as possible, as the ship traversed the track shown. The GEOS-II satellite was tracked by a land-based C-band radar network and also by a C-band radar onboard the VANGUARD. The shipboard tracking was done at the stations TSP1 through TSP8 shown in Figure 1. Astronomical latitudes and longitudes were determined by the shipboard startracker as often as star fixes could be made. A Lorac Long-Range Surface Positioning System (LRSS) was to have provided continuous geodetic location of the ship during the experiment; however, due to equipment malfunction, the LRSS was inoperative throughout the experiment. Therefore, geodetic coordinates from the SRN-9 and GEOS-II observations, in combination with the astronomic coordinates from the startracker, will have to be used to determine the geoidal profile.

In geodesy, it is always desirable to have a traverse or profile start and end on known geodetic stations. In Figure 1, TSP-8 is a known geodetic station at San Juan harbor. TSP-1 at the northern end of the profile is a midocean station of unknown geodetic coordinates. In order to afford the opportunity to make TSP-1 a geodetic control station, it was necessary to emplace on the ocean bottom four underwater acoustic transponders and to measure acoustic ranges from the ship to the transponders while the ship sailed along preplanned tracks around the transponders.[5] The tracks during which useful acoustic data was acquired are shown in Figure 2.

The accuracy of determining the geoidal profile between TSP-1 and TSP-8 will be highly dependent upon the quantity and quality of the SRN-9, GEOS-II and startracker data obtained. Therefore, for the experiment

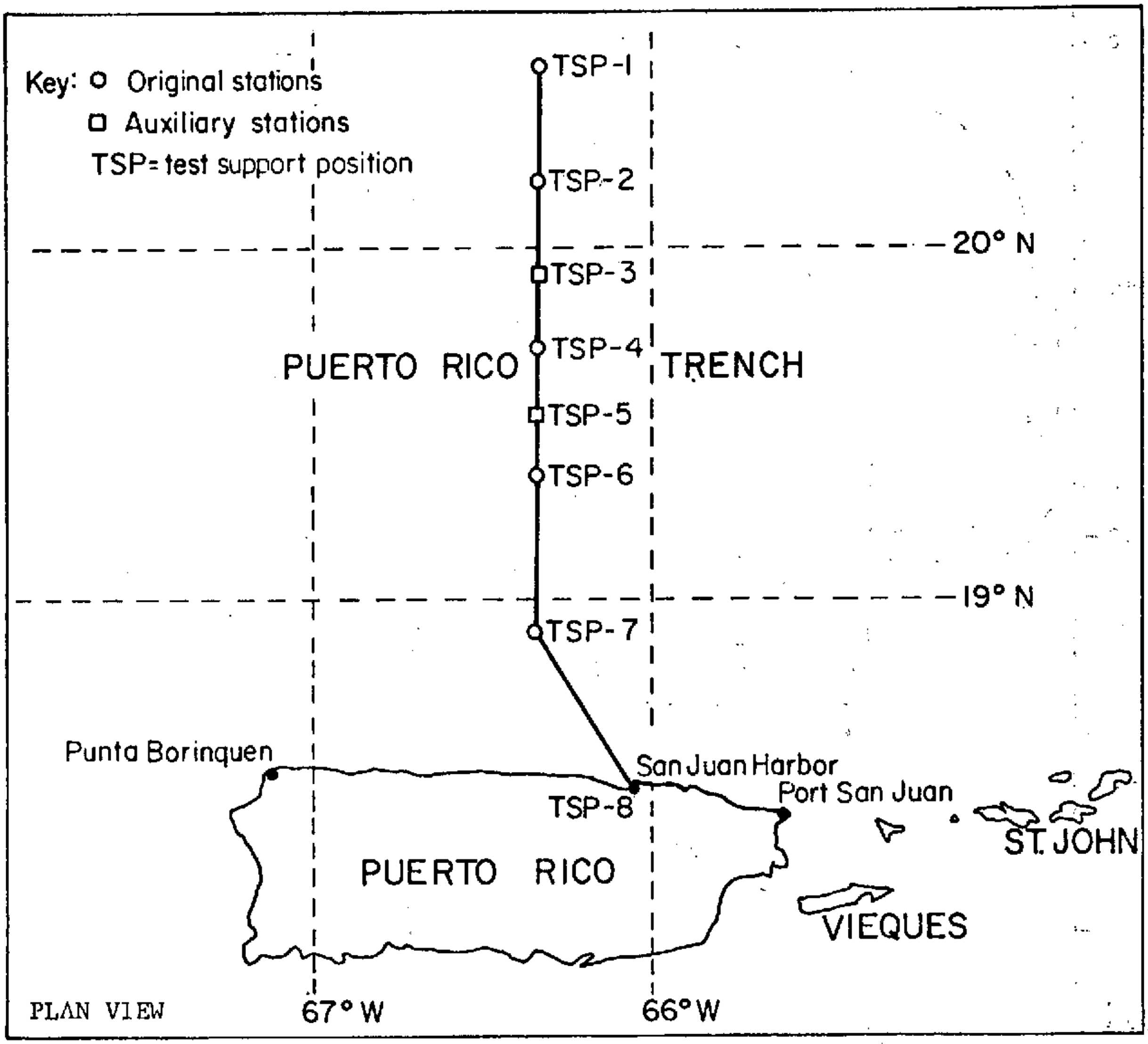

FIGURE 1. SHIP-TRACK TRAVERSE ACROSS THE PUERTO RICAN TRENCH

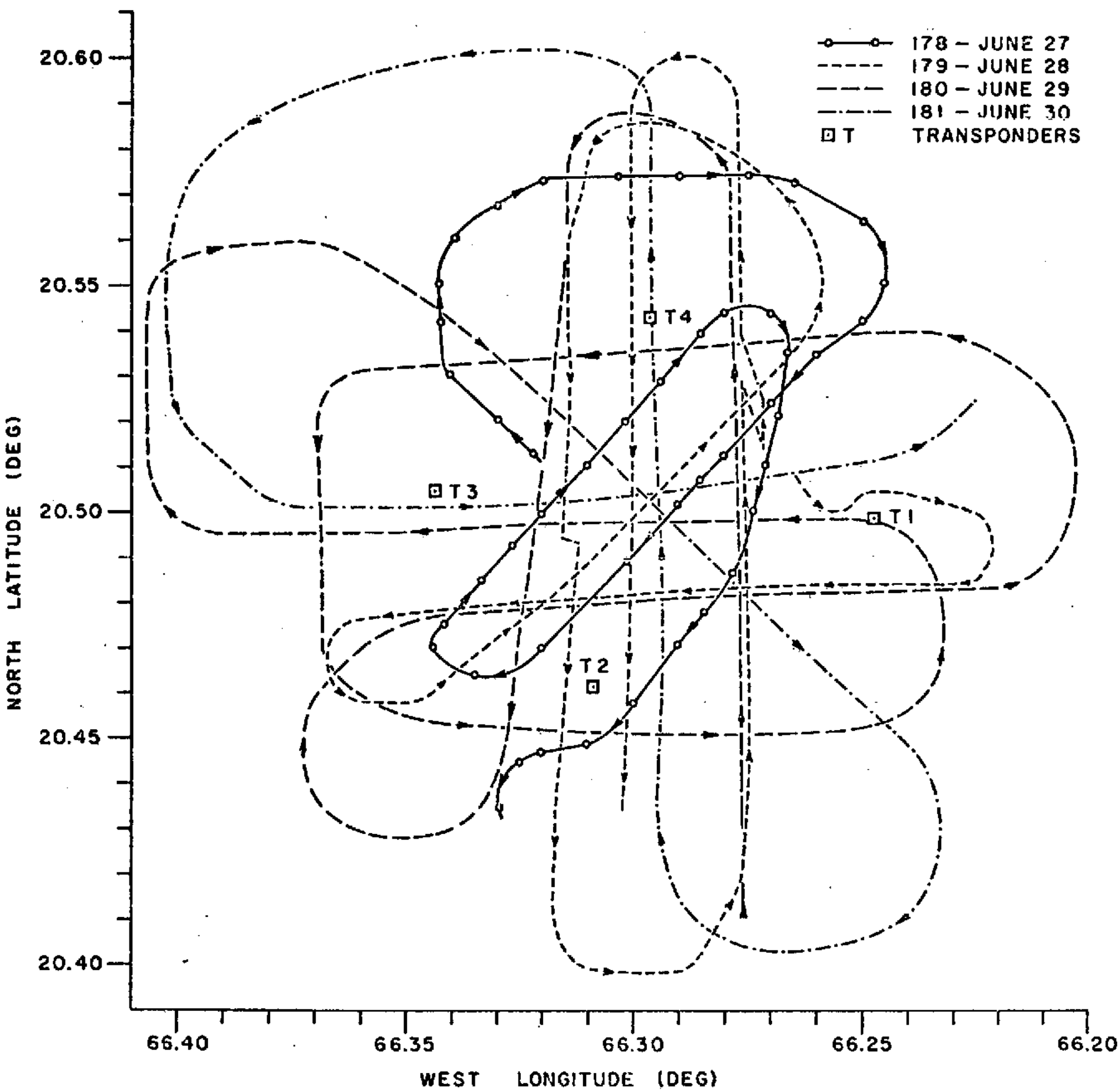

FIGURE 2. SHIP TRACKS DURING SURVEYING OF TRANSPONDERS

to be successful the necessity of having geodetic controls at TSP-1 and TSP-8 is evident for two main purposes: (1) to tie the resultant geoidal profile to a known terrestrial geodetic datum and (2) to use the traverse misclosure which always occurs in practice for geodetic assessment and adjustment of cumulative errors along the entire profile.

Furthermore, after the geodetic coordinates and, hence, the three-dimensional configuration of the transponder network had been defined, other slant ranges from the transponders to the ship could be used to accurately determine the ship's "ground" velocity and geodetic positions at various times. This operation enables assessing the operational performances of the various shipboard navigations systems. The Ship Inertial Navigation System (SINS) of the VANGUARD was a prominent feature in all this work. During the experiment, it furnished a means of navigating the ship along the preplanned tracks. In the absence of data from LRSS and until the SRN-9 and the GEOS-II data are reduced to obtain geodetic coordinates and are incorporated into the analytical processing of data, geographic coordinates and orientation of the network is entirely dependent on the SINS.

Because of the experiences gained in the preliminary examination of the data (before the actual data reduction and analyses) it was necessary to develop two new analytical techniques to accommodate the quality of the measurements. A computer program for each technique was written. These new techniques were then tested successfully with simulated data as described later. This process was extended to include: (1) limited investigations on the selection of optimum spatial distribution of data, (2) optimum magnitude of acoustic ranges relative to transponder depth, and (3) propagation of errors in surface-ship position into the accuracy of geodetic location of transponders.

The two new techniques, LESSA and FIXCOR, developed for geodetic location of transponders are described in Chapter 3. LESSA is for determination of geodetic latitude, longitude and ellipsoidal height (or the corresponding X, Y, Z geocentric Cartesian coordinates) of one or more transponders from surface-ship positions and measured acoustic ranges. It involves parameter weighting to reflect any assessment of errors in surface-ship psotions which, in any other previously published technique, are normally held errorless. FIXCOR is designed primarily for computing the three-dimensional network configuration of arrays of four or more transponders from acoustic ranges coplanar with various pairs of transponders in the array. Its main advantage is that surface-ship-coordinate information is not required unless the true geographic location and orientation of the network is needed. After FIXCOR has been used to determine precisely the geometry of the network, the transponders can be used to track both surface ships and submersibles in the vicinity of the transponders. Thus, the ship's positional changes with time (or velocity) can be deduced.

2.0 SUMMARY OF RESULTS AND RECOMMENDATIONS

Following are qualitative summaries of the simulated-data and field-data results of the experiment and recommendations for: (1) development of methods for utilizing the existing capabilities of the Apollo-type ship instrumentation and satellite technology for geodetic and oceanographic requirements, and (2) identifying and developing new supporting capabilities as required.

2.1 Simulated-Data Results

The simulated data investigation conclusively proved that the newly developed LESSA and FIXCOR techniques are theoretically and practically workable, and effective in meeting the objectives for which they were developed.

In each of the techniques, the system of normal equations was stable. The stability of the normal equations, and hence the accuracy of the determined parameters, are sensitive to spatial distribution of data and the angles of intersection of the acoustic ranges involved. Acute or obtuse angle intersections are not desirable. Optimum ratio of slant range to depth of transponder lies between 1.4 and 1.8. These facts are normal because both techniques are purely geometric solutions. In fact, it is partly because the need for definite knowledge of these conditions was anticipated that the limited simulation study was performed, even though such a study was outside the scope of this task.

For the LESSA techniques, the results of this investigation show that uniform distribution of slant ranges north, south, east, and west of each transponder is necessary for the best accuracy in computed geodetic coordinates of transponders. The absolute geodetic accuracy (that is, relative to the earth's center of mass) of the derived geodetic coordinates is dependent on the absolute accuracy of the surface-ship positions. In other words, the absolute positional accuracy of a tracked object is degraded by the positional errors of the tracking stations, even if the tracking and the tracking system were perfect. Two main features of LESSA are particularly effective. These are: (1) parameter weighting that helps incorporate estimated errors in ship coordinates, and (2) sequential-least-square analysis that adds or subtracts observed data and shows the resultant influence on derived parameters and their variance-covariances.

In addition to the general requirements stated earlier, accuracy from the use of FIXCOR demands that, on the perimeter of the network, there be at least three pairs of noncoincident "coplanar" ranges for each side. The transponder depths need be known only roughly (a priori) to within about ±20 m or less. The technique is highly sensitive to the accuracy of the acoustic ranges upon which the solution entirely depends.

The simulation studies helped greatly in the selection of the most suitable of the myriad of data acquired for determination of the geodetic coordinates of the transponders.

2.2 Field-Data Results

A set of geographic coordinates of each of the four transponders were determined by LESSA. Relative to SINS coordinates of the ship positions the precision achieved ranges from ±12.4 to ±40 m in latitude, ±14.2 to ±25.4 m in longitude, and ±18.3 to ±28.6 m in height. However, relative to an absolute geocentric geodetic coordinate system, the analyses showed that the SINS coordinates were often in error by as much as ±300 m (or about ±10 arc seconds) in both latitude and longitude. The SINS errors were not constant but cyclic and we found that they were also intermittant (step function). Therefore the determination of true coordinates for the control station must await further analysis and the replacement of the SINS data by more reliable and accurate data.

The FIXCOR results which depended only on acoustic ranges were much more reliable. The side lengths were derived with precision ranging from ±5.3 to ±15.5 m, while the precision of each depth was about ±4.5 m. These results are consistent with the quality of data used, as described later.

From the geographic coordinates determined by the LESSA approach, the horizontal side lengths of the network and the depth of each transponder were computed. The FIXCOR program computed the same side lengths and depths independently. The two sets of results closely agreed for the depths only. That the sidelengths showed more variability can be accounted for by the large errors in SINS coordinates, judged from the quality of the data involved. The numberical values are fully discussed in Chapter 5.

2.3 Recommendations

As a result of analysis performed to date of data from the Puerto Rican Trench experiment, the following recommendations are made:

(1) Further analysis should be started immediately so that SRN-9 Doppler satellite data and startracker data can be reduced to compute the geoidal profile between TSP-1 and TSP-8. The SRN-9 data are also required for computation of the absolute location of the transponder at TSP-1. In addition, further analysis is required in order to complete the desired evaluation of other systems on the VANGUARD that were used in the experiment. This would permit other experiments to be made to meet such needs as providing ground-truth information for GEOS-C and other future satellite altimetry missions.

(2) Further analytical investigation and simulation studies should be initiated immediately. This would be the logical extension of the limited simulation investigation that was required to help sift the data acquired and to obtain the quality of results reported here in spite of the various difficulties encountered. The successful completion of further study is critical for improved and economical planning and execution of future marine-geodesy and ocean-physics experiments. Such a study will define requirements for geodetic location when acoustic transponders are used.

(3) The causes of the irregular malfunctioning of the VANGUARD's acoustic system should be determined so that the problems can be rectified.

(4) The operational accuracy of the VANGUARD's star-tracker, which is currently unkwnon, should be established. The resetting or updating of SINS is dependent on the startracker. Future ocean-physics experiments and establishment of ground truth for missions like satellite altimetry require such systems as the startracker. Therefore, the startracker and its interface with SINS

should be thoroughly investigated. The Sperry Kalman Operational Reset (SKOR) for updating SINS also should be scrutinized for possible improvements in computing ship coordinates by SINS.

(5) SINS should not be counted upon in future experiments to furnish the geodetic ship position coordinates because the physical principles upon which it is based align it with respect to the geoid instead of the geodetic reference ellipsoid.

3.0 ANALYTICAL CONSIDERATIONS

This chapter describes the geodetic principles involved and the analytical basis of the data analyses and the computation of the geodetic coordinates.

3.1 The LESSA Program

LESSA is a generalized least-squares-sequential-adjustment computer program written, tested, and adapted for establishing marine geodetic control by transponder location from surface positions. It is an outgrowth of the TLSP (Transponder Location from Surface Positions) technique developed and described in Reference (4). The basis of TLSP is the age-old geometric principle of intersection (i.e., if the positions A, B, and C are known in some coordinate system, the three coordinates required to fix D are determinable by measuring the ranges AD, BD, and CD; see Figure 3). In the experiment, A, B, and C are positions of the interrogating transducer attached to a moving ship. Neglecting the influences of waves and ocean tides, A, B, and C defines a surface parallel to the geoid. In a small area, this surface is approximately a plane.

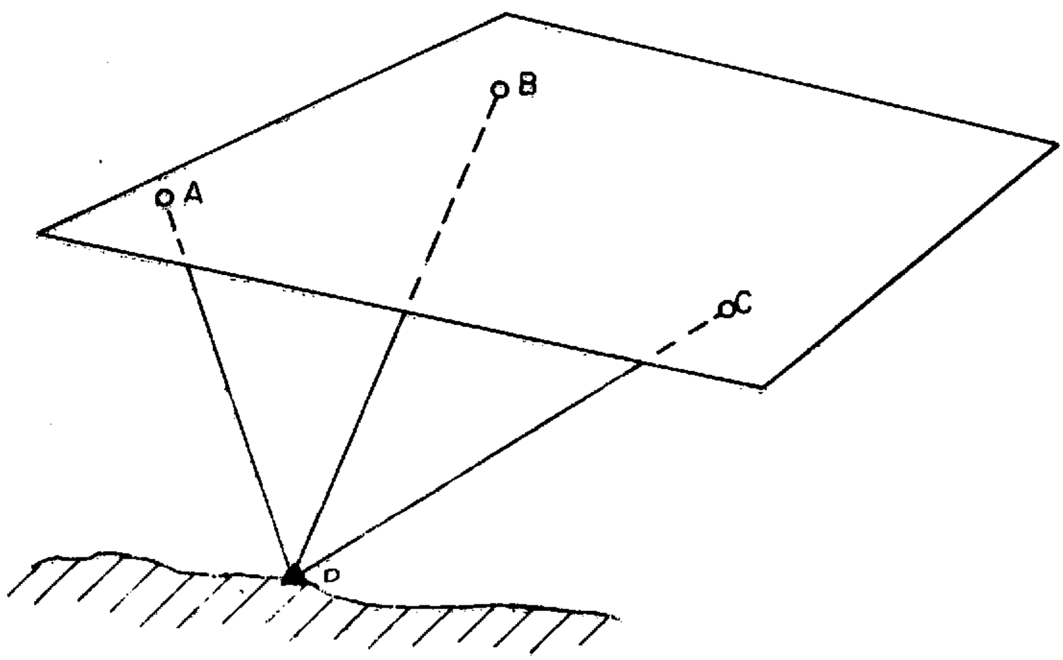

FIGURE 3. THE PRINCIPLE OF TLSP

Transponder interrogation provides the travel time along a refracted ray path between the transducer and the transponder. The TLSP computer program incorporates a ray-tracing subroutine that converts the curved-path travel time to a slant range in linear units. This subroutine requires two inputs: (1) the depth of the transponder (which is yet to be determined) and (2) a sound velocity profile for the area, comprising the speed of sound at various depths. Because of this requirement for depth information, the solution is an iterative procedure. As in all other published techniques, the coordinates of the surface-ship positions are treated as errorless parameters (that is, their variances and covariances are zero). This assumption of errorless parameters is not valid. Therefore, one of the main objectives of LESSA is to avoid this invalid assumption by treating all parameters in such a way that estimates of their variances and covariances are incorporated into the solutions.

The three main features of LESSA which are improvements in the TLSP technique and other similiar previous techniques are:

(1) parameter-weighting which permits more efficient and theoretically rigorous combination and utilization of hybrid data and correct application of error modelling techniques

(2) flexibility in investigating the influences of geometric configurations and spatial data distribution

(3) efficient data editing and updating of previous solutions without repeating previous computations, thereby saving computer time and storage.

An outline of the mathematical formulations follows. It is based on techniques described in References 1, 2, 3, and 6. The functional equation is

$$F_1(X_1^a, X_2^a, L_1^a \;/\; P_1,\ P_2,\ P_3) = 0, \tag{1}$$

from which the following is derived:

$$F_2(\overset{\circ}{X}_1,\overset{\circ}{X}_2,\overset{\circ}{L}_1 \ / \ P_1,\ P_2,\ P_3) + A_1\Delta_1 + B_1\Delta_2 + C_1V_1 = 0, \qquad (2)$$

where:

X_1^a = true values of the ship coordinates

X_2^a = true values of the transponder(s)

L_1^a = true values of the measured slant ranges.

Usually, these true values are not known. Instead, the corresponding approximate values $\overset{\circ}{X}_1$, $\overset{\circ}{X}_2$, and $\overset{\circ}{L}_1$ with associated variance-covariances P_1^{-1}, P_2^{-1}, and P_3^{-1} are estimated or measured. The true and approximate values are related as in:

$$X_1^a = \overset{\circ}{X}_1 + \Delta_1 \qquad (3)$$

$$X_2^a = \overset{\circ}{X}_2 + \Delta_2 \qquad (4)$$

$$L_1^a = \overset{\circ}{L}_1 + V_1 \quad . \qquad (5)$$

A, B and C are the first partial derivatives in a Taylor series expansion of Equation (1), associated with $\overset{\circ}{X}_1$, $\overset{\circ}{X}_2$ and $\overset{\circ}{L}_1$, respectively, while Δ_1, Δ_2 and V_1 are the correction parameters to be determined. Eliminating the lengthy matrix algebra steps in between, it can be shown that the solution of Equation (2) to derive the corrections Δ_1, Δ_2, and V_1 to the assumed $\overset{\circ}{X}_1$ and $\overset{\circ}{X}_2$ and the measured $\overset{\circ}{L}_1$, respectively is

$$\Delta_1 = -N^{-1}A_1^{*}M_1^{-1}W_1 \qquad (6)$$

where * indicates a matrix transpose,

$$M_1 = \left(B_1 P_2^{-1} B_1^* + C_1 P_3^{-1} C_1^*\right) \tag{7}$$

$$N = \left(P_1 + A_1^* M_1^{-1} A_1\right) \tag{8}$$

$$W_1 = F_1 \left(\overset{\circ}{X}_1, \overset{\circ}{X}_2, \overset{\circ}{L}_1\right) \tag{9}$$

and

$$\Delta_2 = P_2^{-1} B_1^* M_1^{-1} \left[A_1 \left(A_1^* M_1^{-1} A_1\right)^{-1} A_1^* M_1^{-1} - I\right] W_1 \tag{10}$$

$$V_1 = P_3^{-1} C_1^* K_1 \tag{11}$$

where

$$K_1 = -M_1^{-1} \left(A_1 \Delta_1 + W_1\right) \quad . \tag{12}$$

The variance factor σ_o is given by either

$$\sigma_o = \left(-K_1^* W_1 / df\right)^{1/2} \tag{13}$$

or

$$\sigma_o = \left[\left(\Delta_1^* P_1 \Delta_1 + \Delta_2^* P_2 \Delta_2 + V_1^* P_3 V_1\right) / df \right]^{1/2} , \tag{14}$$

where

df = number of degrees of freedom.

Hence, the variance-covariance matrices can be shown to be for Δ_1

$$V\Delta_1 = \sigma_o \left[P_1 + A_1^* \left(B_1 P_2^{-1} B_1^* + C_1 P_3^{-1} C_1^*\right)^{-1} A_1\right]^{-1} , \tag{15}$$

for Δ_2,

$$V\Delta_2 = \sigma_o^2 \; P_2^{-1} B_1^* M_1^{-1} \left[I - A_1 \left(A_1^* M_1^{-1} A_1 \right)^{-1} A_1^* M_1^{-1} \right] B_1 P_2^{-1} \quad ; \tag{16}$$

and for V_1,

$$VV_1 = \sigma_o^2 \; P_3^{-1} C_1^* M_1^{-1} \left[I - A_1 \left(A_1^* M_1^{-1} A_1 \right)^{-1} A_1^* \, M_1^{-1} \right] C_1 P_3^{-1} \; . \tag{17}$$

The sequential least squares adjustment with parameter weighting permits the addition of new observations, $\overset{\circ}{L}_2$, (or subtraction of old observations), to update previous solutions and parameter estimates without recomputing previous steps. It may also include estimation of new additional parameters, X_3^a, which are functionally related to the old parameters, X_1^a. These features are effected by the addtion of equations of type

$$F_3\left(\overset{\circ}{X}_1, \; \overset{\circ}{X}_3, \; L_2/P_1, \; P_4, \; P_5 \right) + A_2\Delta_1 + B_2\Delta_3 + C_2V_2 = 0 \quad . \tag{18}$$

Now, let the previous solution for Δ_1 be $\overset{\circ}{\Delta}_1$ to be updated by $\delta\Delta$ as a result of the added new (or removed old) observations. The updated value is now

$$\Delta_1 = \overset{\circ}{\Delta}_1 + \delta\Delta \quad . \tag{19}$$

It can be shown that

$$\delta\Delta = N^{-1} A_2^* \left[A_2 N^{-1} A_2^* + M_2 \right]^{-1} \left[A_2 N^{-1} A_1^* M_1^{-1} W_1 - W_2 \right] , \tag{20}$$

where

M_1 is from Equation (7) ,

N is from Equation (8) ,

W_1 is from Equation (9) ,

$\mathring{\Delta}_1$ is from Equation (6) ,

$$M_2 = B_2 P_4^{-1} B_2^{*} + C_2 P_5^{-1} C_2^{*} \quad ,$$

and

$$W_2 = f\left(\mathring{X}_1, \mathring{X}_3, \mathring{L}_2\right) \quad .$$

In general, the sequential solution results in updated values at the n^{th} sequence of

$$\Delta_n = \mathring{\Delta}_1 + \sum_{i=2}^{n} \delta\Delta_{i-1} \ ,$$

where

$$\delta\Delta_i = -N_{i-1}^{-1} A_i^{*} \left[A_i N_{i-1}^{-1} A_i^{*} \pm M_i\right]^{-1} \left[A_i \mathring{\Delta}_{i-1} + W_i\right], \quad (21)$$

$$N_i^{-1} = N_{i-1}^{-1} A_i^{*} \left[A_i N_{i-1}^{-1} A_i^{*} \pm M_i\right]^{-1} A_i N_{i-1}^{-1} \quad , \quad (22)$$

and the updated variance-covariance matrix is, for Δ_i ,

$$\sigma_o^{\,i} N_i^{-1} \quad , \quad (23)$$

in which

$$\sigma_o^{\,i} = \left[\left(-\ K_i^{*} W_i\right)/df\right]^{1/2} \quad . \quad (24)$$

Similar expressions can be written for Δ_3 and V_2 as in Equations (16) and (17).

These computational procedures can be used in all geodetic adjustments, orbit computations, and data analyses that require rigorous least-squares adjustment techniques. Very often, the ordinary least-squares adjustment (weighted or unweighted) could lead to either unstable normal equations or inability to solve for all the unknown parameters. Often, utilization of the LESSA-type approach, together with the inclusion of effective variance-covariances, eliminates such problems.

3.2 The FIXCOR Program

The Program FIXCOR determines the relative positions of ocean-bottom transponders and the depth of each transponder. There is no necessity to accurately measure each depth directly, and, in any event, this cannot be accurately done without time-consuming ship maneuvering. FIXCOR is a three-dimensional, least-squares solution of intersecting coplanar and/or near coplanar ranges from a ship as it crosses vertical planes containing any two transponders. Figure 4 is a representation of the physical principle of FIXCOR. MSL is the mean ocean surface and S is the position of a ship. R_{iA} and R_{iB} are a pair of coplanar ranges from surface-ship position, S, to transponders A and B whose depths are D_A and D_B, respectively.

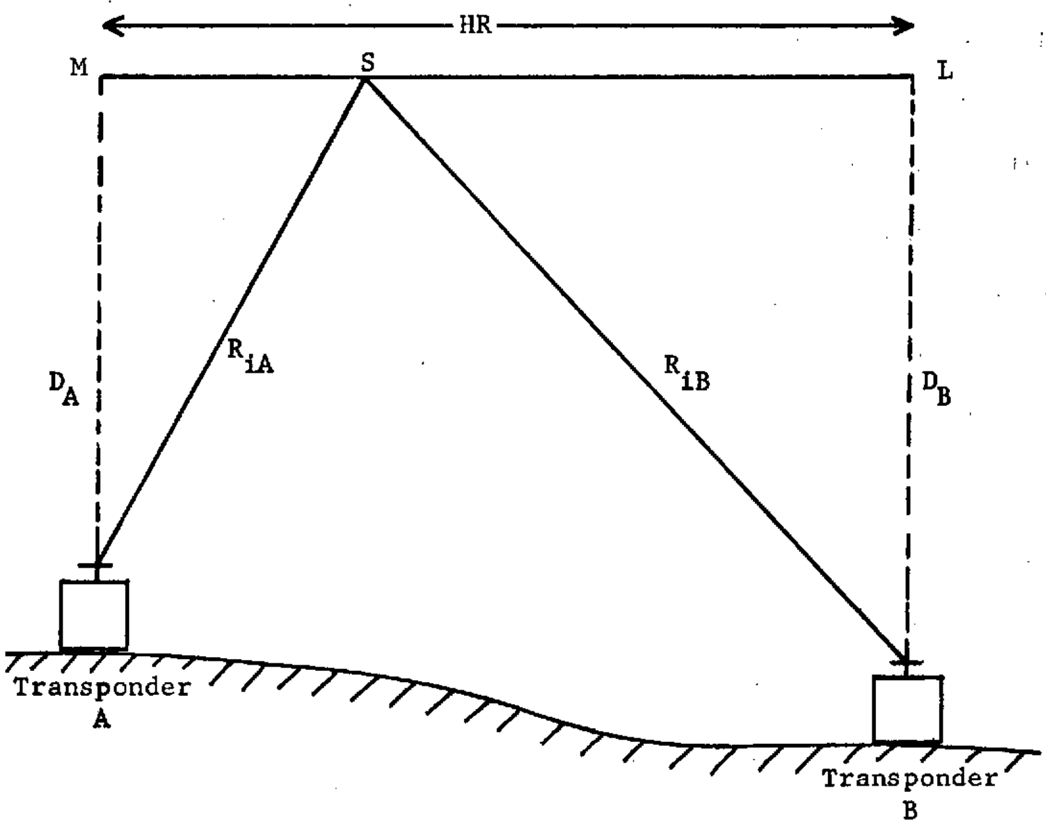

FIGURE 4. VERTICAL PLANE THROUGH TWO TRANSPONDERS

From Figure 4, HR, the sea-level horizontal distance between the two transponders is:

$$HR = \left(R_{iA}^2 - D_A^2\right)^{1/2} + \left(R_{iB}^2 - D_B^2\right)^{1/2} \quad . \tag{25}$$

Given several pairs of R_{iA} and R_{iB}, where $i = 1, \ldots n$, and $n \geq 3$ is the number of pairs of ranges, the unknown HR, D_A and D_B can be uniquely determined. For computing only relative positions and depths, the coordinates of the ship positions do not have to be known. However, to compute the orientation of the transponder array in a known geodetic coordinate system, at least two ship positions of known geodetic coordinates must be used. The results from FIXCOR are easily subjected to statistical testing.

This is a geometricl solution, and, to provide built-in self-checks on the computed results, at least four transponders should be in the network array. The additional advantageous feature of FIXCOR is that the horizontal side lengths and depths of the transponder network can be accurately determined without knowing surface-ship coordinates. The network so defined can be used later to track or monitor surface-ship or other platform movements and compute the speed and heading of such movements.

In applying a least-squares solution to the FIXCOR, the observation equation arising from Equation (25) can be stated simply as

$$V = A\Delta + f(\overset{\circ}{X}) - \overset{\circ}{L} \tag{26}$$

where

V = a vector of residuals, representing the corrections to observed ranges, $\overset{\circ}{L}$.

X^a = true values of HR_{AB}, D_A and D_B shown in Figure 4

$\overset{\circ}{X}$ = an approximate value of X^a

Δ = $X^a - \overset{\circ}{X}$ is the correction to assumed $\overset{\circ}{X}$

A = first partial derivative of $\overset{\circ}{L}$ with respect to $\overset{\circ}{X}$.

The common form of Equation (26) is usually

$$V = A\Delta + W \tag{27}$$

and the solution for Δ is

$$\Delta = -(A^* P_1 A)^{-1} A^* P_1 W \quad , \tag{28}$$

where P is the weight matrix of the observations $\overset{\circ}{L}$. In this form, a successful solution of FIXCOR was found to have stringent requirements relative to the precision of the measured ranges as reflected in P_1 and the closeness of $\overset{\circ}{X}$ to X^a values of the depths.

Consequently, as discussed in the derivation for LESSA in the previous section, the mathematical functional relationship in FIXCOR can be expressed as

$$F \left[(X^{\circ} + \Delta),\ (L^{\circ} + V),\ /P_x,\ P_1 \right] = 0 \tag{29}$$

where P_x is the weighting function associated with X°. The resultant observation equation for a least-squares solution is

$$A\Delta + BV + W = 0 \tag{30}$$

and the solutions for Δ and V are given by

$$\Delta = - \left[P_x + A^*(BP_1^{-1}B^*)^{-1}A \right]^{-1} A^*(BP_1^{-1}B^*)^{-1}W \tag{31}$$

and

$$V = - P_1^{-1}\ B^*(BP_1^{-1}B^*)^{-1}\ (A\Delta + W), \text{ respectively.} \tag{32}$$

The variance of unit weight σ_o is, as usual, given by

$$\sigma_o = \left(\left[(BP_1^{-1}B^*)^{-1}\ (A\Delta + W) \right]^*\ W/df \right)^{1/2} . \tag{33}$$

The variance-covariance matrix for the adjusted parameters $(X^{\circ} + \Delta)$ is

$$\sigma_o \left[P_x + A^*(BP_1^{-1}B^*)^{-1}A \right]^{-1} , \tag{34}$$

which is used to assess the quality of the derived parameters. This type of approach eliminates the problem in the use of the method as expressed in Equations (27) and (28)

4.0 PRE-ANALYSIS USING SIMULATED DATA

Before applying the new FIXCOR program to field data, it was necessary to test it with simulated data. The LESSA program also required testing with simulated data.

The advantage in using simulated data is that one selects absolute values for the parameters and generates the true values of the corresponding observations. These observations can then be randomly perturbed by introducing errors whose magnitudes and signs are exactly known. Using such perturbed data in the computations, one derives, for the required parameters, values that can then be compared with the initial absolute values selected. Such simulations demonstrate whether the analytical techniques are workable in practice and if they are, to what accuracy and under what conditions.

4.1 LESSA Test Using Simulated Data

The LESSA simulation studies, in addition to establishing operational capability, were extended to include three limited investigations. These covered: (1) how surface-ship coordinate errors propagate into the coordinates of deduced transponder coordinates, (2) the optimum distribution of surface-ship positions around each transponder, and (3) the influence of geometric configuration as a function of the ratio of slant-range lengths to transponder depth. These investigations were limited because they were outside the scope of this contract. They were necessary to provide results to serve as guidelines in the selection of data required for this report from the multitude of acoustic ranges obtained during the experiment. Despite the limitations, the results were invaluable for obtaining the best possible accuracy in determining the transponder locations from the type of data discussed in Chapter 5.

Since this is not a report on simulated studies, only the important results that relate to the objectives of the task are presented. Table 1 and Figure 5 show some of the results obtained from the simulation investigations using the analytical procedures outlined in Section 3.1. Modeling the errors of surface-ship coordinates is achieved through the weighting functions P_2 and P_4 of Equations (2) and (18). The data in Table 1 show that when errors that should be modeled are not, the determined transponder coordinates (Column 2) could be seriously in error while, ironically, the precision estimates from the weight-coefficient matrix (Column 3) give very optimistic but erroneous results. The determined transponder coordinates (Column 4) are more nearly identical to the true values simulated. The corresponding precision indicated by the weight-coefficient matrix (Column 5) is poor and reflects the fact that the absolute accuracy of determined transponder coordinates cannot be better than the accuracy of surface-ship positions.

Figure 5 shows the influence of geometric configuration. The effective test parameters are the angles at which slant ranges from surface-ship positions intersect at the ocean-bottom-mounted transponder. This is defined by the ratio of the slant range to the depth, which ratio must always be greater than 1. When this ratio is 1 or nearly 1, the ship is almost vertically over the transponder. When the ratio is exactly 1, the transponder coordinates are indeterminate by least-squares estimate. However, in practice, the position fix is obtainable although inaccurate. The amount of error is the sum of the ship's position errors due to the ship not being exactly over the transponders and errors due to poor resolution from ranges intersecting at acute angles. As the ratio increases to and beyond 1.9, the accuracy of the transponder fix deteriorates again, due to large obtuse intersection angles or poor geometry. The optimum geometry is obtained when the ratio is between 1.4 and 1.8.

TABLE 1. EFFECT OF EFFICIENT ERROR MODELING[a]

	Recovery of True Coordinates of Transponder in Meters			
	Ship Position Errors Not Modeled[b]		Ship Position Errors Modeled[c]	
Transponder Coordinates	Absolute Error of Recovery of Transponder Coordinates	Precision of Recovery as Indicated by Weight Coefficient Matrix	Absolute Error of Recovery of Transponder Coordinates	Precision of Recovery as Indicated by Weight Coefficient Matrix
X	-22.8	±3.6	-1.9	±20.5
Y	-3.0	±2.1	-0.2	±20.0
Z	12.6	±3.3	1.4	±20.1

(a) The simulations involved the following errors:

Ship Positions - ±20 m in each of X, Y, Z

Slant Ranges - ±3 x (a function of Slant Ranges).

(b) "Not modeled" assumes errorless ship coorinates.

(c) "Modeled" takes into account errors in ship coordinates.

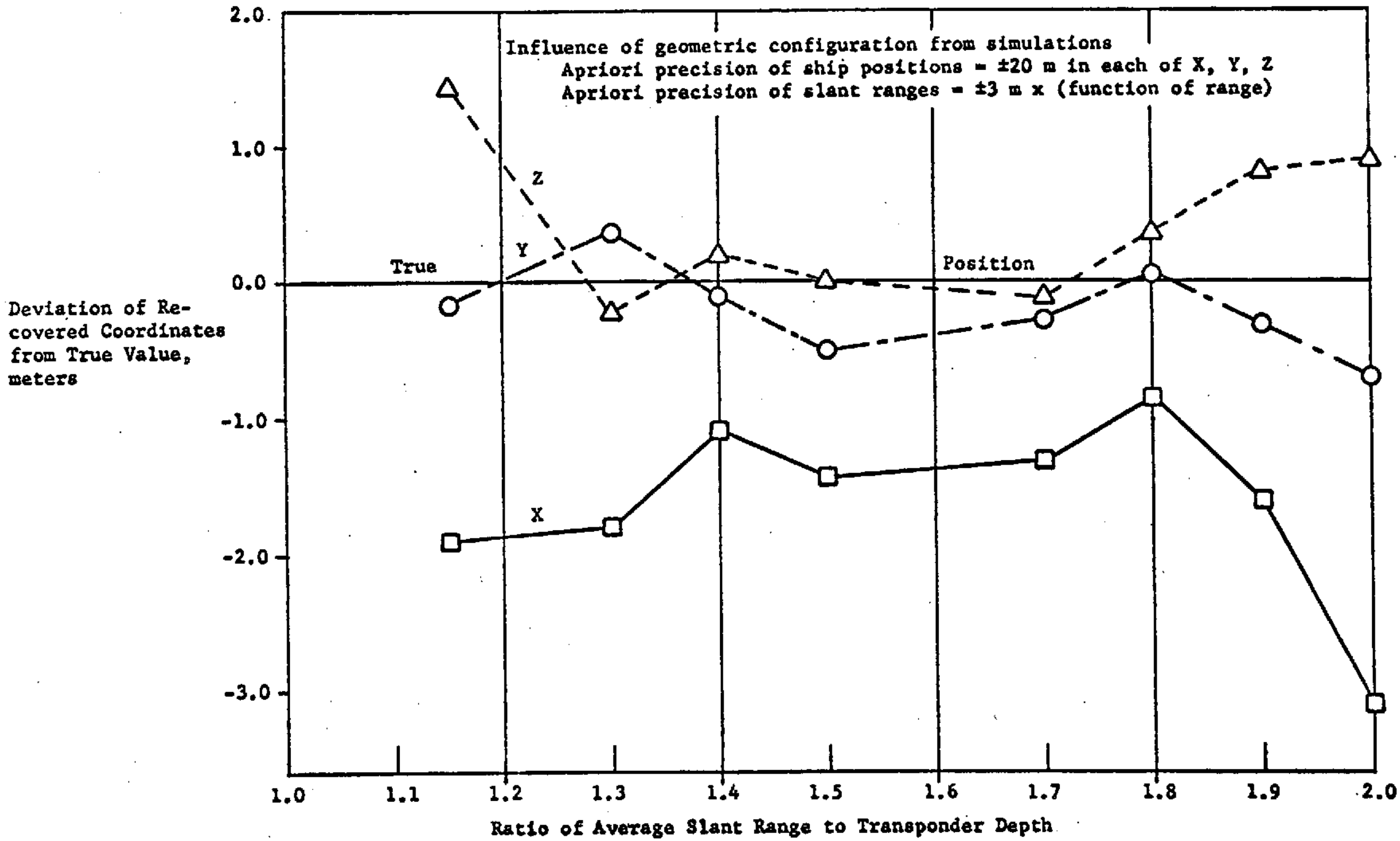

FIGURE 5. INFLUENCE OF GEOMETRIC CONFIGURATION IN LESSA PROGRAM

The ten simulation-data points in each case were uniformly distributed on the perimeter of squares centered at the transponder. Uniform data distribution around the transponder was found to give the best results in each of the cases considered. None of the sets of results shown in Figure 5, for the recovery of X, Y, Z coordinates, lies on a uniform curve with unique minima and maxima. The reason is that the solution, which is purely geometric, is very sensitive to spatial distribution of data. As a result, a smooth curve can be expected only if the data points are distributed not on squares but on circles centered over the transponder, and also that corresponding points on each square lie on straight lines radiating from the ocean-surface point centered over the transponder. However, there is no doubt that, below the ratio 1.3 and beyond the ratio 1.9, indeterminancy of results must be expected. Besides, beyond the 1.9 ratio, the ranges become so oblique that the influence of acoustic refraction may become unpredictable or sound-channel influence may become a factor. Furthermore, in practice, collecting data on a circular distribution pattern is not realistic. Therefore, no simulation was undertaken on circular-type patterns.

4.2 FIXCOR Test Using Simulated Data

During this test, a network of four transponders was simulated, as for the network of the field experiment. The network was a quadrilateral that was not exactly a square or any other type of parallelogram. The aim was to simulate reality.

FIXCOR was proved to work effectively, generally in two to three iterations. Networks of various size were used. For each, a rough initial estimate of the transponder was obtained by adding errors of various magnitude to the initial exact value to be recovered from the solution.

The smaller the network, the less the error that can be accommodated in the a priori estimate of the transponder depth. The one thing that FIXCOR cannot tolerate is large errors in measured slant ranges. The ideal results were found for the case when the sides of the quadrilateral were 11 to 12 km and the average depth about 5 km. These results are shown in Tables 2 and 3. The standard errors (square root of each main diagonal element of the variance-covariance matrix) of the derived parameters were of about the same magnitude as the average random errors introduced into the simulated slant ranges. For this network, inaccuracy in the rough depth estimates ranged from 20 to 100 m. Smaller networks could not tolerate more than 20-m error in rough estimates of depth.

In general, the success of the FIXCOR program requires that certain conditions be satisfied concerning: (1) the total number, n, of pairs of coplanar ranges, (2) the number, ℓ, of pairs of coplanar ranges for each side of the network polygon, (3) the precision of each acoustic slant range, and (4) the approximate depth of each transponder. Conditions (1) and (2) as in the original FIXCOR program require that

$$2\,n \geq s + t + n \tag{1}$$

and

$$\ell \geq 3 \tag{2}$$

where

n = s x ℓ = total number of pairs of coplanar ranges

ℓ = number of pairs of coplanar ranges per side

s = number of sides for the transponder network

t = total number of transponders.

TABLE 2. FIXCOR DETERMINATION OF TRANSPONDER DEPTHS[a]
Values in Meters

Transponder Number	Rough Estimate of Depth	True Value of Depth	Depth From FIXCOR	Standard Error[b]
1	4860	4960.5	4958.4	±2.3
2	5080	5060.5	5057.8	±2.6
3	4980	5026.5	5024.7	±2.2
4	5170	5120.5	5118.9	±2.2

(a) In the simulations, the corresponding groups of ranges were noncoplanar to simulate reality in which it is near impossible to obtain exactly coplanar ranges.

(b) The standard error estimates include the precision estimate of ±3 meters assigned to the slant ranges. This accounts for the apparently large values of standard errors that are bigger than the absolute errors, i.e., the differences between Columns 3 and 4.

TABLE 3. FIXCOR DETERMINATION OF TRANSPONDER ARRAY SIDE LENGTHS
Values in Meters

Side	Side Length Horizontal	Standard Error
1-2	11,501.4	±3.3
1-3	17,303.7	±2.1
2-3	12,935.6	±3.0
3-4	11,492.0	±3.1
4-1	12,938.2	±2.9
4-2	17,307.7	±2.2

Condition (3) can accommodate ranges that have systematic errors of constant magnitude but not those of varying magnitudes of more than 3 m unless at least two transponder depths are known with sufficient accuracy to hold them fixed in a least-squares solution. Condition (4) often requires that the depths be approximately known to within ±20 m unless the slant ranges are very accurate and the side length to depth ratio is large.

A much more extensive and exhaustive investigation is necessary before conclusions of this type can be considered as universally valid.

5.0 FIELD-DATA REDUCTION AND ANALYSES

Of the various field data obtained during the experiment, those relevant to the objective of this work are: (1) the acoustic slant ranges from the ship to the transponders; (2) surface-ship coordinates as derived from (a) SINS, (b) SRN-9 Doppler satellite receiver and (c) shipborne C-band radar observation of GEOS-II satellite. The coordinate information from sources (2b) and (2c) are currently not available. Therefore, the computations and analyses discussed in this report involve only data from sources (1) and (2a) above.

In situ acoustic velocity was not measured at the time of the experiment. Various velocity-profile data for the area were obtained from the National Oceanographic Data Center. Since the various velocity-profile data were measured at different seasons of the year, the most consistent set available for the season of the experiment was selected for use. That profile is shown in Figure 6.

5.1 Editing and Selection of Acoustic Data

Throughout the experiment, each transponder either gave good results intermittently or did not function at all during certain periods. These results were provided in a previous Battelle report to NASA, Reference (5), and are reported in Section (5.3), Tables 8 and 9. Earlier, preliminary examination of the data had, of course, indicated this unaccountable intermittent malfunctioning of the VANGUARD's acoustic

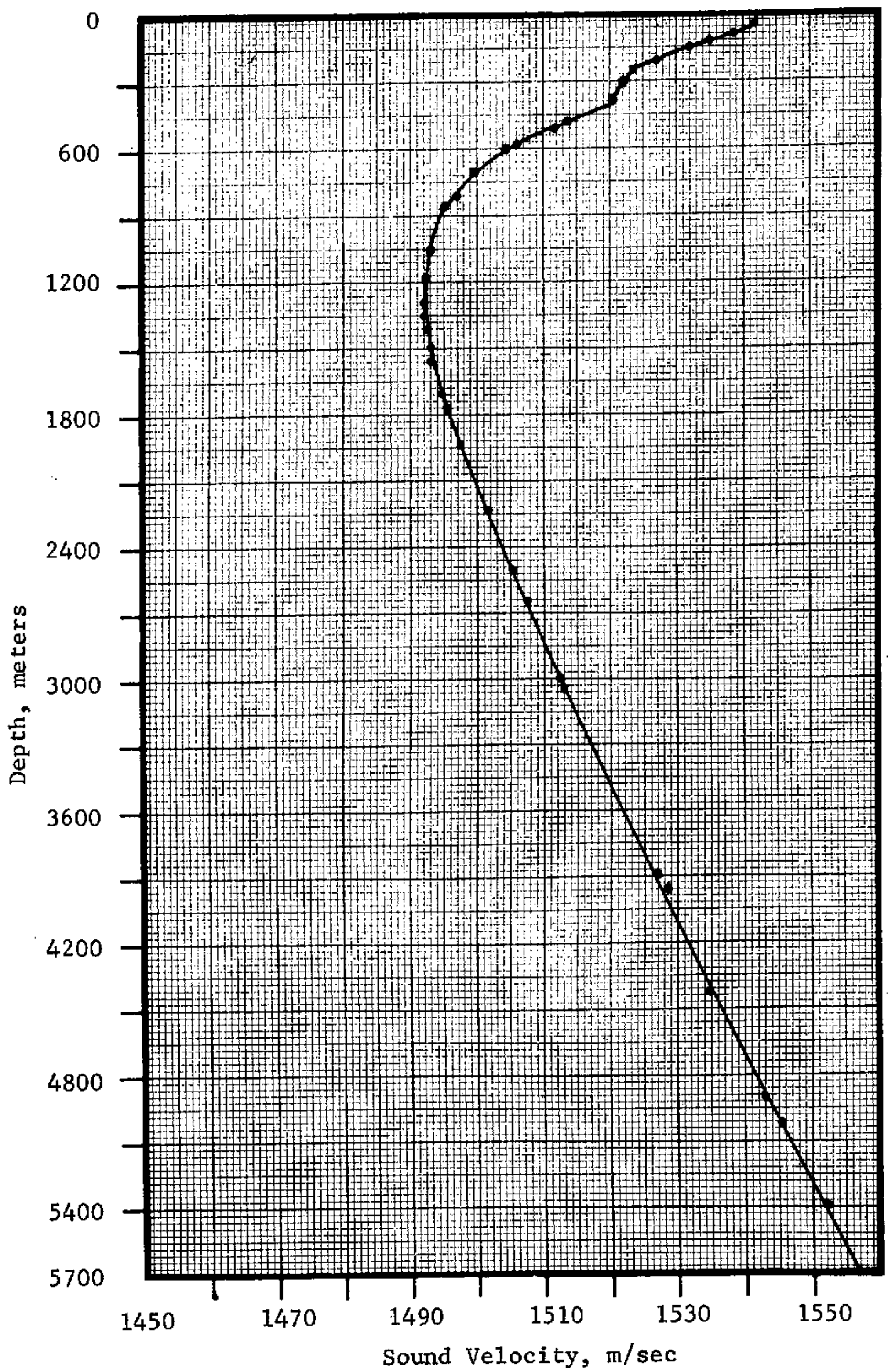

FIGURE 6. VELOCITY OF SOUND PROFILE IN THE PUERTO RICO TRENCH AREA (November 24, 1961)

system. As a result, prior to the receipt of the acoustic data, a limited simulation study was conducted to give guidelines about the optimum selection of data in both quantity and spatial distribution.

The computer printout showed that the valid data from each transponder was not only intermittent but also that they occured at highly irregular time intervals. The data density was once every 13 seconds on some days and once every 19 seconds on others. From one time interval of a valid set of data to another, extensive extrapolations and interpolations were made to identify the next set of valid data. This operation was further complicated by the fact that, for each transponder, the rate of change of recorded slant ranges varied from about 1 to 40 m per interrogation interval (13 or 19 sec). This variation depended on: (1) the ship's speed which was variable; (2) the heading of the ship, which varied from track to track daily; and (3) the distance of the ship's track from the transponder.

For these reasons, a considerable portion of the data were first visually edited before the selected data were subjected to automatic editing by computer. The data editing involved three main stages. The first stage was to extract all the valid data from the irregular and intermittent response of each transponder. The other two stages involved selection of data suitable for the FIXCOR and the LESSA programs discussed previously.

5.1.1 Selection of Data for FIXCOR

The acoustic ranges required for FIXCOR are those measured at the various instants when the ship's acoustic transducer was in any of the vertical planes containing any pair of transponders (see Figure 4). These instants are not usually known exactly. First, from Figure 2 and from the computer printout of the acoustic records already edited, data were selected for the time periods when the ship tracks crossed any of the relevant six vertical planes of the network. An array of four transponders as in Figure 2 has six vertical planes containing the various pairs of transponders.

The only time periods that could be selected were those during which the ship's heading was approximately constant. The data for each crossing were passed through a polynomial curve fitting routine to determine the time for the minimum of the curve. The pair of ranges corresponding to that time were selected as the required coplanar ranges. The results of this selection are shown in Table 4. From the distribution of each set of data around its mean curve and from the rate of change of the acoustic slant ranges per interrogation time interval, relative standard deviation was estimated for each range. The data in Table 4 are based on the raw records from the field.

5.1.2 Selection of Data for LESSA

The visual and the automatic data editing eliminated all acoustic ranges that had obvious gross errors. From the remainder, data sets had to be selected for each transponder to meet, as much as was possible, the "optimal" conditions deduced (Section 4.1) for high accuracy in the

TABLE 4. COPLANAR ACOUSTIC SLANT RANGES IN YARDS (RAW DATA)

Zulu Date/Time[(a)]	Ranges to Transponders	
	LINE 1-2	
	#1	#2
179/11:37*	7981 ±30	6421 ±5
179/3:44	6780 ±20	7360 ±10
180/15:31	7106 ±23	7000 ±16
	LINE 1-3	
	#1	#3
181/6:43*	7970 ±30	7556 ±30
181/2:59	7778 ±3	7533 ±2
179/5:36	7706 ±25	7645 ±3
179/11:24	7476 ±20	7829 ±2
180/13:24	9019 ±5	6594 ±5
178/8:09	7720 ±1	7597 ±1
178/10:46	6861 ±3	8622 ±5
179/7:22	8938 ±13	6640 ±10
179/9:29*	6297 ±36	9603 ±8
178/9:24*	9394 ±42	6542 ±19
	LINE 1-4	
	#1	#4
179/5:51	7121 ±15	6643 ±5
178/7:45	7021 ±1	6731 ±2
178/10:16	7034 ±8	6720 ±5
178/9:42*	8701 ±12	5912 ±9
	LINE 2-3	
	#2	#3
179/5:16	6644 ±4	6664 ±1
180/13:39	6404 ±20	6997 ±20
180/15:05	6600 ±16	6729 ±10
179/4:09	6728 ±8	6581 ±5
179/7:34*	6350 ±14	7110 ±18
	LINE 2-4	
	#2	#4
179/5:34	7744 ±1	7743 ±1
179/3:57*	6552 ±10	9334 ±10
	LINE 3-4	
	#3	#4
179/7:10	7200 ±9	6585 ±8
178/4:52	6480 ±1	7308 ±1

(a) The sets marked with * were found to contain gross errors when used in FIXCOR, and therefore were not used in the final computations.

geodetic determination of ocean-bottom transponders. Because the only surface-ship position information available at this stage of the work was from SINS, it was necessary to select data only from the periods when the ship was either not turning or had completed turning and maintained fairly uniform heading and speed for about an hour.

5.2 Computed Parameters and Comparative Analyses of Results

The VANGUARD's acoustic system at the time of the experiment operated at a built-in acoustic velocity of 1463.04 m/sec. Before the computations, all the selected acoustic data had to be processed through a ray-tracing program as described in Reference (4). The ray-tracing program plays a dual function. First, it computes from sound-velocity-profile data the sound velocity for the area and corrects each range for the systematic error due to the built-in sound velocity used. Second, in the same process, ranges are corrected for acoustic refraction as necessary.

The quality of data selected for FIXCOR did not meet all the conditions deduced in the simulation studies described in Section 4.2. For this reason, the analytical procedures stated in Equations (29) through (34) were developed and used in the FIXCOR computation. The Equations (1) through (24) were used in the computations of the data set selected for LESSA determination of the transponder coordinates.

The results from the FIXCOR program are shown in Tables 5 and 6. The standard errors (the square root of each main diagonal element of the variance-covariance matrix defined in Section 3.2) varied from ±5.3 to ±15.5 m for the computed sidelengths. The maximum error was in Side 2-4 which had only one pair of usable coplanar ranges instead of the ideal three-pairs minimum. Such precision indicators are consistent with the type of data involved in the computations and are as expected from the simulation studies. It should be noted that the network side lengths are small relative to the average depth, whereas the simulation investigation showed that FIXCOR obtains optimum results when the ratio is about 2 to 2.5.

Table 7 shows the geographic coordinates and the ellipsoidal height of each transponder as deduced from the LESSA computations. During the experiment, Transponder 1 gave the most erratic response and it also turned out to have the worst kind of data distribution. Not surprisingly, the standard errors show that its coordinates are the least precise for the four transponders. The coordinates are based on latitude and longitude output from SINS. Because SINS coordinates are not "true" geodetic coordinates, the transponder coordinates so derived are also not true geodetic coordinates. Further, the large errors in SINS coordinates also propagated into transponder coordinates derived from SINS.

TABLE 5. SEA-LEVEL HORIZONTAL DISTANCES BETWEEN TRANSPONDERS

Side	Length, Meters	Standard Errors, Meters
1-2	7172.2	±6.8
2-3	5697.1	±14.8
3-4	6435.9	±5.3
4-1	6418.1	±7.6
1-3	9135.3	±9.6
2-4	9364.1	±15.5

TABLE 6. COMPUTED TRANSPONDER DEPTHS
(in meters)

Transponder	Mode of Computation and Precision			
	FIXCOR		LESSA	
	Depth	Standard Error	Depth	Standard Error
1	5663.8	±3.7	5668.7	±28.6
2	5683.0	±4.6	5691.3	±24.3
3	5630.0	±4.2	5638.4	±19.2
4	5682.5	±5.3	5709.3	±18.3

TABLE 7. THREE DIMENSIONAL GEOGRAPHIC COORDINATES OF TRANSPONDERS

Transponder	Latitude North			Longitude West			Ellipsoidal Height, meters
	o	'	"	o	'	"	
1	20	29	53.84	66	14	51.34	-5720.7
2	20	27	39.96	66	18	31.37	-5743.3
3	20	30	15.50	66	20	36.28	-5690.4
4	20	32	33.47	66	17	46.64	-5761.3

From the sizes of the residuals and the variances from the FIXCOR solution, there is no doubt that the parameters derived from it are precise to within ±5 m. However, since FIXCOR gives only relative positions, it will be necessary to incorporate geodetic ship position coordinates from either the SRN-9 Doppler satellite solutions or GEOS-II C-band radar data solutions before further comparisons and evaluation of results can be accomplished.

6.0 REFERENCES

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APPENDIX

Tables Developed Under Previous Contract

Tables A-1 and A-2, developed under a previous Battelle contract (see Reference 5) provide information relevant to this report.

TABLE A-1. ACOUSTIC DATA ACQUISITION TIME
(Hour of Day: 0^h-24^h)

Zulu Day No. and Date	Duration of Entire Run	Individual Transponder Data Acquisition Time 1	2	3	4
Day 178 6/27/70	4:49-6:25	4:49-4:55	NIL	4:49-5:53	4:49-6:25
	6:27-8:00	7:11-8:00	NIL	7:38-8:00	6:27-8:00
	8:02-9:34	8:02-8:39	NIL	8:02-9:34	8:02-8:37 9:09-9:34
	9:36-11:07	9:37-11:07	NIL	9:36-9:50 10:02-10:06 10:09-11:07	9:36-11:07
	11:09-12:09	11:10-11:42	NIL	11:09-12:09	11:09-11:19
Day 179 6/28/70	2:01-3:25	2:04-2:38 2:42-3:25	NIL	2:01-2:43	2:01-2:53
	3:26-5:01	3:27-4:11	3:26-5:01	3:38-5:00	3:30-4:14
	5:02-6:34	5:16-5:52 5:56-6:21	5:02-5:52	5:02-5:56	5:15-6:34
	6:35-8:06	7:00-7:34	7:07-8:06	6:48-7:58	6:35-7:36
	8:10-9:40	8:49-9:17 9:24-9:40	8:10-9:40	9:13-9:40	9:16-9:40
	9:43-11:16	10:57-11:11	11:10-11:16	9:44-9:48 10:53-11:16	9:43-10:28
	11:18-12:00	NIL	11:18-12:00	11:18-11:57	NIL
Day 180 6/29/70	10:40-12:11	NIL	11:04-11:41	11:04-11:45	NIL
	12:13-13:46	12:40-13:46	12:43-13:46	12:40-13:46	NIL
	12:47-15:20	NIL	12:47-15:20	12:47-15:20	NIL
	15:22-16:48	15:22-16:48	15:25-15:59	15:22-15:39	NIL
	17:17-18:13	17:18-17:39	17:30-17:49	17:17-18:13	NIL
	18:29-20:03	19:42-20:03	18:29-20:03	18:29-19:41	NIL
	20:22-21:11	20:22-20:33 21:05-21:11	NIL	NIL	NIL
	21:50-22:27	NIL	21:51-22:16	21:50-22:27	NIL
	22:47-23:17	NIL	NIL	22:47-23:17	NIL
Day 180/181 6/30/70	23:35-0:57	NIL	NIL	23:35-0:57	NIL
	2:27-3:35	NIL	NIL	2:27-3:35	NIL
	5:34-7:00	6:31-7:00		5:34-7:00	NIL

TABLE A-2. TRANSPONDER RESPONSE, QUALITY, AND QUANTITY OF MEASURED RANGES

Track Number	Transponders -- (Range in Yards)			
Zulu Day (1970)	1	2	3	4
1 Day 179	NIL	40 minutes of data. Good PCA. Range 8600-6000	As for 2. Ranges 10,250-7800	NIL
2 Day 179	Intermittent response. 4 minutes of useable range 10,250-7100	Intermittent for 50 minutes. Good PCA. Range 10,000-6600	53 minutes of nearly continuous response. Good PCA. Range 10,000-6700	78 minutes. Intermittent during initial 30 minutes. Very good on the curve close by. Good PCA. Range 10,200-6600
3 Day 180	NIL	22 minutes data. Good PCA. Range 8000-7400	30 minutes data. Good PCA. Range 9300-5900. Good data outside quad	NIL
4 Day 180/181	NIL	NIL	70 minutes data. Good PCA. Range 10,200-6800	NIL
5 Day 178	4 minutes of data. No PCA. Range 9800-10,300	NIL	60 minutes data. Good PCA. Range 6500-10,200	93 minutes of data. Data outside quad very good. Good PCA. Range 7300-8000
6 Day 181	6 minutes intermittent data. Quality questionable. Range 9800-6700	NIL	72 minutes good data and PCA. Range 5900-10,200	NIL
7 Day 178	46 minutes intermittent data. Reliable PCA. Range 10,300-7000	NIL	20 minutes good data. Range 10,300-8300	92 minutes good data all around. Range 7100 to 8300 to 7000. Good PCA
7a	32 minutes intermittent Range 7300-10,000, 10,200-8800. No PCA	NIL	90 minutes. Good PCA. Range 8100-6800-7100-6000-7600	56 minutes. Range 7300-10,200-6300
7b Day 178	20 minutes intermittent 1 or 2 PCA good. Range 8600-8800-6700	NIL	70 minutes intermittent. No PCA. Range 8000-10,200-8300	87 minutes. Good PCA. Range 6100-5900-8300

TABLE A-2. TRANSPONDER RESPONSE, QUALITY, AND QUANTITY OF MEASURED RANGES
(Continued)

Track Number Zulu Day (1970)	Transponders -- (Range in Yards) 1	2	3	4
8 Day 181	NIL	NIL	53 minutes intermittent. Good PCA. Range 10,000-7500	NIL
9		NO RECORD	RUNS NOT CONDUCTED DURING EXPERIMENT	
10		NO RECORD		
11 Day 179	2 minutes. Questionable data and PCA. Range 6100-7500	70 minutes intermittent. Good PCA. Range 9400-6200-9200-7300	83 minutes. Good data and PCA. Range 10,200-6400-8000-7300	43 minutes. Good data and PCA. Range 10,300-9300-10,200
12		NO RUN RECORD		
13 Day 179	8 minutes intermittent data. PCA fair. Range 1000-8900	40 minutes good data and PCA. Range 10,100-6000-7700	70 minutes good data and PCA. Range 10,200-6600-10,300	60 minutes good data and PCA. Range 7400-6100-10,200
14		NO RUN RECORD		
15 Day 179	3 minutes intermittent data	62 minutes intermittent data. Good PCA. Range 8500-7000-9800	24 minutes good data and PCA. Range 10,200-9600-9800	18 minutes intermittent data. Range 10,200-7100
16	RUN No. 16 NOT MADE. But track connecting Run #5 and Run #7 (Day 178) is approximately Run #16 in opposite direction			
A Day 180	NIL	20 minutes intermittent data. Range 8300-7300-10,000	31 minutes good data and PCA. Range 10,400-9800-10,300	NIL
B I Day 180	12 minutes intermittent data. Questionable PCA. Range 10,300-9000-10,000	20 minutes intermittent. No PCA but good line 2-3 crossing. Range 10,400-6100	53 minutes good data and PCA. Range 11,000-6600-7400	NIL

TABLE A-2. TRANSPONDER RESPONSE, QUALITY, AND QUANTITY OF MEASURED RANGES (Continued)

Track Number	Transponders -- (Range in Yards)			
Zulu Day (1970)	1	2	3	4
B II Run B Cont'd into Run C Day 180	Few minutes of questionable data	52 minutes intermittent data. 2 good PCA. Range 6000-8700-6300	73 minutes mostly good data. PCA is fair. Range 7700-10,000-6600-7800	NIL
C Day 180	Few ranges may be retrievable by interpolation and extrapolation	16 minutes of questionable data	12 minutes of fairly good data	NIL
D Day 180 and portion of E	9 minutes useable but intermittent data. Range 7100-10,000	12 minutes. Good PCA. Range 10,200-10,100	47 minutes good data and PCA. Range 10,200-6600	NIL
E Day 180	4 minutes of intermittent poor data	64 minutes good data and PCA. Range 8800-6100	59 minutes good data. Range 6600-10,200	NIL
Unscheduled Runs Day 180	NIL	NIL	19 minutes fairly good	NIL
Day 178	6 minutes	NIL	54 minutes	9 minutes
Day 179	23 minutes	NIL	42 minutes	55 minutes